Control of chaos by random noise in a system of two coupled perturbed van der Pol oscillators modeling an electrical discharge plasma

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The control of chaos in nonlinear systems by different methods is still a high interest topic particularly when this is achieved by random noise as in this work. The change of chaotic dynamics into periodic dynamics induced by random noise in a system of two coupled perturbed van der Pol oscillators and comparison with the experimentally observed behavior of a double discharge plasma that it models is presented. Methods specific to nonlinear analysis such as phase portraits, Lyapunov exponents, and Fourier spectra are used to demonstrate the changeover from chaotic to regular dynamics induced by random noise. A phase diagram determines the range of noise parameters corresponding to the lowest orders of an observed bifurcation sequence of 3×2^n type and particulars of the transitions are presented.

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I. INTRODUCTION

The presence of noise is a highly complicating aspect in both the theoretical and experimental analysis of the dynamics of nonlinear systems. The interplay between noise and chaotic behavior is very intricate and sometimes, as in the case of $1/f$ noise, the two notions could not be separated even conceptually.

In theoretical studies on dynamical systems described by differential/difference equations, the effect of noise is relatively easily studied because the behavior is obtained through computation with and without the noise terms. For some particular situations, the addition of noise can have predictable results. Any effect that occurs at some well-defined value of the control parameter will be smeared out by the noise that manifests itself as small fluctuations in the control parameter, especially for observations of the attractor characteristics on scales smaller than the noise level [1,2]. In particular, this limits the possibility of observing high bifurcation orders in bifurcation sequences [3]. If noise is present, then a system that is close but not yet in a crisis can be "bumped" into and out of the crisis region by the noise [4,5]. In intermittency, noise can change the time between bursts by orders of magnitude [6].

The correlation dimension $C(R)$ calculated could be affected by the presence of noise. If the amplitude of noise is σ, then we expect the noise to dominate for $R<\sigma$, where R is the distance from a given point such that all the attractor is within a radius R. Since noise is supposedly random, the noise-dominated data will tend to spread out uniformly in a domain of the state space and we expect to find a correlation dimension equal to the state space dimension for small values of R [1,2]. The noise tends to make the slope of the $\ln C(R)$ vs $\ln R$ larger for small values of R.

In real nonlinear physical systems, the situation is still more complicated. The dynamic behavior of the system is obtained by carrying out measurements of characteristic parameters against a constant background of noise that hampers measurement and corrupts the data. Separating the contribution to the data of the intrinsic dynamics and of the random noise is an extremely tricky problem. Even giving an answer to the question: is the irregularity (nonperiodicity) of the data due to nonlinear determinism or rather due to random inputs to the system or random fluctuations of the parameters is no trivial matter.

Various testing algorithms for discerning between the nonlinearity and linear stochastic behavior have been proposed [7–9]. Also, different methods of analysis have been devised [10–14] aiming to replace noisy measurements by better values that contain less noise. From the previous consideration, the noise appears as a nuisance to the dynamics physicist. However, this point of view has slightly changed during the last decade since the pioneering work of Ott, Grebogi, and Yorke [15] that stimulated considerable interest of the control of chaos and synchronization by various perturbations applied to the system including random noise [16–24].

The present work is a study of the effect of random noise on a system of two perturbed coupled van der Pol oscillators modeling a double electrical discharge plasma. Comparison between the experimental data and the results of the ideal, noiseless model is quite good for almost all the range of the control parameter used in the study [25] with the exception of a small interval. The addition of noise terms to the ideal equations extends the agreement to the entire range of the control parameter covered in this work.

Subsequently, the study is extended to additional noise-induced dynamics consisting of the control of chaos for the particular system of two perturbed coupled van der Pol oscillators, mainly to an induced bifurcation sequence of 3×2^n type. Details of the transition from one period to the next are presented.

II. EXPERIMENTAL STUDY

The experimental setup was presented in detail in Refs. [25,26]. It consists of a system allowing simultaneous functioning of two electrical discharges sustained by separate voltage sources. They are running in the same discharge tube filled with argon at low pressure. The anodes of the two discharges are situated facing each other at a distance of a few centimeters. They are biased one against the other by a continuous voltage source whose voltage U_m is considered as the experimental control parameter. An additional coupling is possible by connecting in series with the biasing source a sinusoidal voltage U_e with amplitude $(5-10)$ % of the continuous biasing. Without this forcing the discharge shows periodic dynamics with a fundamental frequency that slowly changes with changing of the control parameter.

The global behavior of the system as a function of the continuous biasing and in the presence of the sinusoidal signal with a frequency in the neighborhood of 2.5 times the free oscillation frequency shows intervals of regular and chaotic dynamics as well as transitions to chaos by different mechanisms. This behavior is correlated to the characteristics of the space charge structures such as double layers (DL) [27].

It was observed that the position and the characteristics of one or more space charge structures generated in the interanode space depend on U_m, on the discharge currents I_1, I_2, and also on the gas pressure p. For certain values of the amplitude and frequency of U_e, quasistationary plasma formations can appear [28]. In this work, the amplitude of U_e is kept constant so that it can be considered as generating a forcing regime.

Experimental investigations of the current-voltage characteristics, I vs U_m, correlated with plasma potential and light intensity distributions, which reveal the formation of DL, are extensively presented in Refs. [28,29]. It is also shown that for values of a DL potential drop larger than the ionization potential of the working gas, the structure becomes unstable and the current I has periodical oscillations.

In this work we present the influence of noise on phenomena taking place in the interanode space in the presence of both the continuous biasing and the sinusoidal forcing.

III. COMPUTATION MODEL

In Ref. [25] we proposed a model based on physical arguments similar to those considered in Ref. [29]. The two discharge plasmas that generate space charge structures in the interanode space are considered as independent van der Pol oscillators with a special type of coupling and strength determined by the control parameter denoted as m.

The biasing dc and ac potentials are modeled as separate terms. The periodic perturbation of amplitude e and frequency g is introduced in the conventional way. Consequently, we consider the dynamics of the following five equations system that differ from that given in Ref. [25] by the presence of the noise terms:

$$\dot{x}_1 = x_2 + m x_4 + D_1 \xi_1,$$

$$\dot{x}_2 = -c(x_1^2 - 1)x_2 - x_1 + e\cos x_5 + m x_4 + (m-n)x_3 + qm,$$

$$\dot{x}_3 = x_4 + -m x_2 + D_2 \xi_2,$$

$$\dot{x}_4 = -f(x_3^2 - 1)x_4 - x_3 - e\cos x_5 - m x_2 - (m-n)x_1,$$

$$\dot{x}_5 = 2\pi g.$$

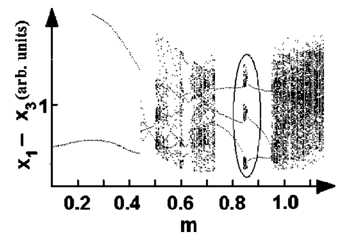

FIG. 1. Bifurcation diagram for the interanode current, modeled by $x_1 - x_3$.

The incorporated additional noise terms are $D_1\xi_1$ and $D_2\xi_2$. Here ξ_i $(i=1,2)$ are produced by a random noise generator used to provide a random real number uniformly distributed in a prescribed interval whose magnitude will be denoted A and called "the noise range." This number is multiplied by a constant D intended to set the maximum amplitude of the noise and to give a measure to the standard deviation σ. The correlation is of δ type, namely, $\xi(t)\xi(t+\tau) \sim A^2\delta(\tau)$.

The study in the absence of noise $(D_1=0, D_2=0)$, presented in Ref. [25], shows that computational reproducing of the three experimental elements: (i) time series, (ii) phase portraits, and (iii) fast Fourier-transform spectra requires a very precise tuning of the model control parameter m. The plot of the values of m vs the corresponding values of the continuous biasing potential U_m shows a linear dependence. This result was interpreted as demonstration of the validity of the model.

However, in a certain interval of values of the continuous biasing potential, the experimental results were in apparent contradiction with the results of the model. This situation is easily understood on the basis of the bifurcation diagram for the interanode current modeled by $x_1 - x_3$. This diagram, shown in Fig. 1, represents the global dynamics of the system without noise.

In the region enclosed within an oval in the figure, corresponding to an U_m interval of 0.5 V around 33 V, experimental data show no discontinuity in the period three dynamics. A blow up of this region of the bifurcation diagram is shown in Fig. 2.

The noiseless system of equations represents an idealized situation. To make it more realistic, we considered that the noise, always present in an electrical system, has to be reflected by the model. The origin of noise is related to the

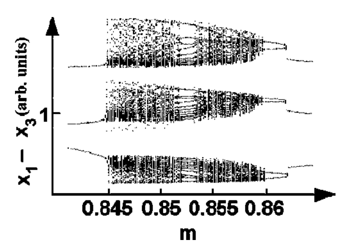

FIG. 2. A blowup of the region of the bifurcation diagram enclosed within an oval in Fig. 1.

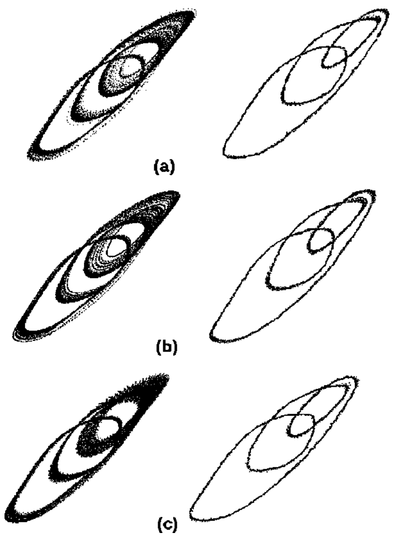

FIG. 3. Phase portraits showing the noise induced transition from chaotic behavior to period three dynamics for (a) $m=0.847$, (b) $m=0.850$, (c) $m=0.857$; left column, without noise; right column, with noise characterized by $D=0.25, A=0.85$.

fluctuations generated by the thermal motion of the ambient electrons and ions [30].

In this study we consider only the case of identical noise functions, $D_1\xi_1=D_2\xi_2$.

For convenient values of the noise parameters D and A, we demonstrate that, for any value of m in the interval of chaotic dynamics around 0.850, the irregular dynamics is transformed into period three dynamics.

In this way, the noise rehabilitates the model making it generate an output in agreement with the experiment for the whole range of control parameter considered. The values of the noise parameters in the corresponding interval can be interpreted as a measure of the real noise present in the experimental system.

The analysis of the system dynamics is carried out using the following methods: phase portrait, Lyapunov exponents, and Fourier spectrum.

The noise-induced transition from chaotic behavior to period three dynamics is illustrated in Fig. 3, which presents phase portraits for three arbitrary values of the control parameter in the chaotic region around $m=0.850$, namely, $m=0.847$(a), $m=0.850$(b), and $m=0.857$(c). The graphs on the left correspond to the noiseless behavior (chaos or very high periods) and the graphs on the right show the period three dynamics induced by a noise characterized by $D=0.25$, $A=0.85$. Each of these diagrams is represented using the same number of points (3800).

The sampling velocity (number of points) was optimized by taking 50 points for a period of the fundamental oscillation of the system, without forcing ($e=0$) and for $m=0.850$. It is important to mention that in all the computed results presented in this work, the first 5000 points of the integration process were eliminated in order to allow the system to overcome the transients.

For the narrow window in the neighborhood of 0.853 where the dynamics of the noiseless system is regular, the noise with the same characteristics induces lower period dynamics similar to the chaotic zones.

TABLE I. Noise-induced change of the Lyapunov exponent for three arbitrary values of m, the same as in Fig. 3.

		Lyapunov exp.
$m=0.847$	Without noise	0.022 ± 0.015
	With noise	0.011 ± 0.019
$m=0.850$	Without noise	0.085 ± 0.041
	With noise	0.009 ± 0.018
$m=0.857$	Without noise	0.051 ± 0.018
	With noise	0.012 ± 0.029

Besides the phase portraits, we tried to detect the noise-induced effects by a Lyapunov exponent analysis. Standard considerations on the optimization of the reliability of this analysis [31,32], and the relationship between the characteristic frequencies and the time step in our computation, led us to consider that a sequence length of 3000 points is optimum.

The results are given in Table I, which presents the maximal (average) Lyapunov exponent with noise and without noise for the chosen values of the control parameter. The noiseless values are definitely positive, indicating chaotic dynamics. When random noise characterized by $D=0.25$, $A=0.85$ is applied, the Lyapunov exponents become zero within the precision of the computation algorithm, indicating periodic dynamics.

The same result is reflected in the change of the Fourier spectra shown in Fig. 4. Here, we consider only the value of m in the middle of the interval ($m=0.850$) because similar change is characteristic for the whole interval. The spectrum on top of the figure (a) corresponds to the chaotic behavior shown by the noiseless system. The second spectrum from the top (b) is characteristic of the system in the presence of random noise with parameters in the interval that induces period three dynamics.

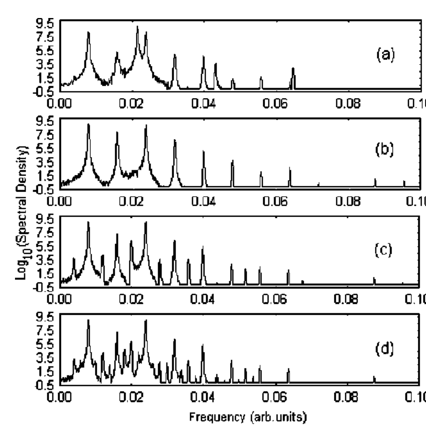

FIG. 4. Noise-induced changes of the Fourier spectra for $m=0.850$: (a) without noise; (b) with noise characterized by $D=0.25$, $A=0.8$ (period three); (c) with noise for $D=0.25$, $A=0.5$ (period 3×2); (d) with noise for $D=0.25$, $A=0.35$ (period 3×2^2).

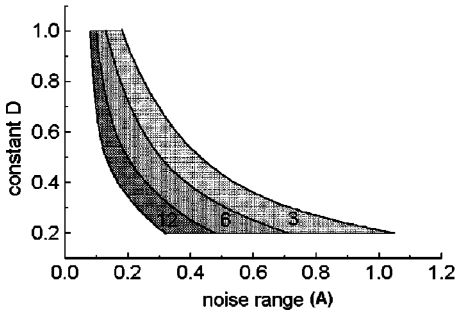

FIG. 5. Phase diagram showing the change in the dynamics of the system induced by the noise for the values of D and A corresponding to relevant situations.

We observe that the noise has the following effects: it annihilates the parasitic frequency (and its harmonics) that appears by the splitting of the forcing component, changes the ratio of the amplitudes, reduces the amplitude of the chaotic background, and induces the generation of higher frequency harmonics.

It is interesting to observe that the noise in the range of parameters that brings the model into agreement with the experiment for values of the control parameter m around $m = 0.850$ has no influence on the dynamics of the system for m outside this interval; the system is robust to noise outside this interval.

IV. NOISE-INDUCED BIFURCATION SEQUENCE

Figure 5 presents a phase diagram showing the change in the behavior of the system induced by the presence of noise. The value of the control parameter is again $m = 0.850$.

For noise characterized by the values of the parameters D and A in domain (3), the experimental behavior (period three) is regained. In the domain denoted (6) on the figure, period 3×2 is induced and for the parameters in domain (12), period 3×2^2 is found.

The bifurcation sequence for period three is clearly illustrated by the spectra in Fig. 4 going in the downward direction. The second top spectrum, (b), corresponds to period three dynamics obtained for an added noise characterized by $D = 0.25$, $A = 0.85$. Keeping D constant and reducing the noise range A, we obtain period 3×2 dynamics for A in the neighborhood of 0.5 to which the third spectrum from top (c) corresponds. Further reducing A, we identified period 3×2^2 in the neighborhood of $A = 0.35$ with the characteristic spectrum shown at the bottom of Fig. 4.

The next bifurcation could hardly be obtained in a very small interval of A and the following bifurcations were impossible to observe because the added noise becomes too small to influence the system dynamics.

Outside the shaded area in Fig. 5, chaotic dynamics or very high period is observed; a very low noise does not influence the behavior while a high noise considerably perturbs the system adding up to the already chaotic dynamics.

The noise level that generates the effects shown on Fig. 5 is in a range of 10% of the amplitude of the sinusoidal perturbation and in a range of 5% of the amplitude of $x_1 - x_3$.

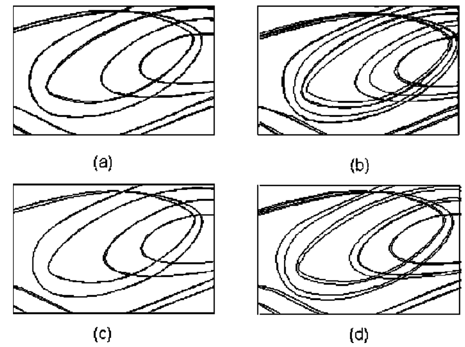

FIG. 6. Details of phase portraits showing the transition between neighboring dynamics: (a) and (c) period 3×2; (b) and (d) period 3×2^2. The noise parameters are $D = 0.22$, $A = 0.455$.

It is important to mention that the transition from one type of regular dynamics to the next is not as clearly defined as shown by the diagram. Between two neighboring zones there is an interval of values of the noise range in which the system goes almost periodically from one dynamics to the other. This is easily observed if the integration procedure is carried on for long enough [33].

In Fig. 6, the transition from period 3×2 to 3×2^2 and back to period 3×2 is illustrated by details of the two phase portraits at four times. By proper choice of the noise parameters D and A we manage to obtain a situation where the time that the system persists in each of the two dynamics is approximately equal to about 2500 points. The transition interval looks like a very noisy period three and lasts for about the same time. This was achieved for $D = 0.22$ and $A = 0.455$. The four situations shown in Fig. 5 correspond to an interval of about 2000 points centered on the following times (in number of points): $t(a) = 1350$, $t(b) = 6400$, $t(c) = 11\,350$, $t(d) = 16\,400$.

V. CONCLUSIONS

In the first three sections of the work it is demonstrated that the addition of noise terms to the system of equations modeling a double discharge plasma improves the model, extending the agreement with experiment to the whole range of values of the experimental control parameter studied.

Particularly, we show that the added random noise with certain characteristics reestablishes the agreement in a domain of the control parameter where the experiment shows period three dynamics and the noiseless equations show chaotic behavior, without destroying the agreement corresponding to other values of the control parameter. Clearly, the situation is equivalent to a control of chaos by random noise.

We extended the study of this aspect by changing the characteristics of the added noise. Ranges of parameters were identified corresponding to dynamics of 3×2^n type. Unlike the suddenness of bifurcations caused by the variation of the control parameter, in the case of bifurcations induced by noise variation there exists a small but definite range of noise parameters for which an almost periodical transition from one stage of bifurcation to the next is taking place.

This bifurcation sequence could not be obtained in experiment because, as seen from Fig. 5, higher bifurcation orders correspond to lower noise levels and we did not yet manage to reduce the intrinsic noise of the discharge system without changing the other dynamics characteristics. If this could be achieved, the noise could be controlled by using an external noise generator.

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